NOV 1964 ATSS-10 Library

National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-3760

NASA TTF-8986

ST - AD - AMP - 10235

N65-20777 (ACCESSION NUMBER)	1 (THRU)
7 (PAGES)	(CODE)
(NASA CR OR TMX OR AD NUMBER)	33 (CATEGORY)

FACILITY FORM 602

UNSTEADY FUSION OF BODIES UNDER THE EFFECT OF AERODYNAMIC HEATING

by

S. K. Matveyev

[USSR]

GPO PRICE $ ________

OTS PRICE(S) $ ________

Hard copy (HC) $1.00

Microfiche (MF) $.50

4 NOVEMBER 1964

NASA TT-F-8986
ST — AD — AMP — 10235

UNSTEADY FUSION OF BODIES UNDER THE EFFECT OF AERODYNAMIC HEATING

Vestnik Leningradskogo Universiteta
No. 13, Seriya matematiki, mekhaniki
i astronomii, vyp. 3, pp. 159 - 162,
LENINGRAD, 1964

by C. K. Matveyev

1. - FORMULATION OF THE PROBLEM

Assume that a body, subject to aerodynamic heating, begins to melt. The forming liquid film on the melting surface moves under the effect of pressure gradient and surface friction. The equations of motion of the liquid film are equivalent to the boundary layer equations.

We shall consider the flow at moments of time near the beginning of fusion. We thus estimate the film movement as laminar and we shall neglect the inertial terms in the equation expressing the law of the quantity of motion. We shall neglect in the equation of energy the heat from the work of friction forces and the deceleration temperature will be estimated to be approximately equal to the thermodynamic one.

In the system of coordinates, linked with the body surface at the time $t = 0$ (x being counted along this surface and y along the normal to it in the depth of the body), the equations of motion will take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1,1}$$

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \rho X - \frac{\partial p}{\partial x} = 0, \tag{1,2}$$

$$\rho c \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \lambda \frac{\partial^2 T}{\partial y^2}. \tag{1,3}$$

* NESTATSIONARNOYE OPLAVLENIYE TEL POD DEYSTVIYEM AERODINAMICHESKOGO NAGREVA.

where u, v are the velocity components; T is the temperature; X is the component of mass forces in the direction of the axis x; μ and λ are respectively the viscosity and heat conductance factors; c is the heat capacity; ρ is the density; p is the pressure. We consider X, p and consequently $\frac{\partial p}{\partial x}$ as well known functions of x; ρ, c and λ are considered constants

Initial conditions:

$$\text{at } t=0 \quad u=v=0, \quad T=T_0, \quad y_w=0. \tag{1,4}$$

Boundary conditions:

$$\text{at } y=y_w(x,t) \quad \lambda\frac{\partial T}{\partial y}=-q_w, \quad \mu\frac{\partial u}{\partial y}=-\tau_w. \tag{1,5}$$

q_w and τ_w are the thermal flow and friction. The quantities with the index „w" refer to the surface of the liquid film.

In the absence of vaporization and of heterogenous reactions, the surface of the liquid film is determined by the equation

$$\frac{\partial y_w}{\partial t}+u_w\frac{\partial y_w}{\partial x}=v_w. \tag{1,6}$$

For a semi-infinite body made of material devoid of a clearly-expressed melting point, we have the conditions

$$\text{at} \quad y\to\infty \quad T\to T_\infty, \tag{1,7}$$

$$\text{and at} \quad T=T_m \quad u=v=0. \tag{1,8}$$

Most of the researchers ([1], [2], [3] and others) impose for a semi-infinite body made of glass-like material the following condition instead of (1.8):

$$\text{at} \quad y\to\infty \quad u\to 0, \quad v\to 0. \tag{1,8'}$$

Such formulation of the problem is incorrect, for, integrating the equation (1.2) under that condition, we arrive at the integral

$$u=\int_\infty^y \frac{1}{\mu}\left[\left(\frac{\partial p}{\partial x}-\rho X\right)(y-y_w)-\tau_w\right]dy,$$

which has a finite value only in the case, when $\mu\to\infty$ at $T\to T_\infty$.

Such disparity of conditions (1.8^1), correct at first sight, and of the equation (1.2) is due to the fact, that glass-like materials reveal elastic properties at low temperatures, and the stress tensor has a form distinct from the stress tensor of a viscous fluid.

The accounting of this phenomenon is quite complex, and it is thus appropriate, as was pointed out in [4], to introduce a conditional "point of fusion" – T_m, approximately determining it as such, at which the elastic properties of the material become noticeable. The authors, setting up the conditions (1.8^1), are anyhow practically compelled to either take the finite limits of integration, which correspond to choice of T_m, or to limit themselves in selecting T_∞, as is done by Satton [1], who takes $T_\infty = 0^\circ$ K.

2. TRANSITION TO LAGRANGE VARIABLES.

The solution of the problem formulated is made more complex by the fact that the boundary conditions are assigned on a mobile surface, whose position and shape are not known beforehand. In connection with this, a transition to Lagrange variables offers interest, since their boundary will be fixed.

We shall integrate the equation (1.2) over y:

$$\mu \frac{\partial u}{\partial y} = \left(\frac{\partial p}{\partial x} - \rho X\right)(y - y_w) - \tau_w, \tag{2.1}$$

and we shall then pass to Lagrange variables ξ, η, t, linked with the liquid particles, i. e. we shall observe the variation of hydrodynamic quantities in the particle (see the diagram Fig. 1, next page). Then

$$x = x(\xi, \eta, t), \quad y = y(\xi, \eta, t).$$

In order to find x and y as functions of ξ, η, t, we have the equations

$$\frac{\partial x}{\partial t} = u, \quad \frac{\partial y}{\partial t} = v \tag{2.2}$$

with the initial conditions:

$$\text{at } t = 0 \qquad x = x_0(\xi, \eta), \quad y = y_0(\xi, \eta). \tag{2.3}$$

The selection of $x_0(\xi,\eta)$ and $y_0(\xi,\eta)$ is up to us; in the particular case we may have $t=0$ $x=\xi$, $y=\eta$.

In order to pass to variables ξ,η,t in the equations (2.1) (1.3), we shall make use of the formulas :

$$\frac{\partial}{\partial\xi}=\frac{\partial}{\partial x}\frac{\partial x}{\partial\xi}+\frac{\partial}{\partial y}\frac{\partial y}{\partial\xi},$$
$$\frac{\partial}{\partial y}=\frac{\partial}{\partial x}\frac{\partial x}{\partial\eta}+\frac{\partial}{\partial y}\frac{\partial y}{\partial\eta},$$
$$\frac{\partial}{\partial t}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}.$$

At moments of time near the beginning of fusion, and at appropriate choice of x_0, y_0, we may estimate $\frac{\partial x}{\partial\eta}$ and $\frac{\partial y}{\partial\eta}$ of one order, and since we have in the boundary layer

$$\left|\frac{\partial u}{\partial x}\right|\ll\left|\frac{\partial u}{\partial y}\right|,\ \text{then}\quad \frac{\partial u}{\partial\eta}=\frac{\partial u}{\partial y}\frac{\partial y}{\partial\eta}$$

Similarly $\frac{\partial T}{\partial\eta}=\frac{\partial T}{\partial y}\frac{\partial y}{\partial\eta}$.

Passing to Lagrange variables in the equation (2.1) and integrating it over η, we shall have

$$u=\int_{\eta_m}^{\eta}\frac{1}{\mu}\frac{\partial y}{\partial\eta}\left[\left(\frac{\partial p}{\partial x}-\rho X\right)(y-y_w^*)-\tau_w\right]d\eta \tag{2.4}$$

($\eta=\eta_m(\xi,t)$ is the line where $T=T_m$).

The continuity equation in Lagrange variables has the form

$$\frac{D(x,y)}{D(\xi,\eta)}=\frac{D(x_0,y_0^*)}{D(\xi,\eta)}$$

Differentiating this equation in time and utilizing (2.2), we obtain

$$\frac{\partial u}{\partial\xi}\frac{\partial y}{\partial\eta}+\frac{\partial x}{\partial\xi}\frac{\partial v}{\partial\eta}-\frac{\partial u}{\partial\eta}\frac{\partial y}{\partial\xi}-\frac{\partial x}{\partial\eta}\frac{\partial v}{\partial\xi}=0.$$

Assuming that $\frac{\partial x}{\partial\eta}$ and $\frac{\partial y}{\partial\eta}$ are of one order, we may neglect the term

$$\frac{\partial x}{\partial\eta}\frac{\partial v}{\partial\xi}$$

In such case we shall obtain from the last equation by integrating over

$$v=\int\limits_{\eta_m}^{\eta}\frac{\frac{\partial u}{\partial\eta}\frac{\partial y}{\partial\xi}-\frac{\partial u}{\partial\xi}\frac{\partial y}{\partial\eta}}{\frac{\partial x}{\partial\xi}}\,d\eta.$$

The equation (1.3) is transformed to the form

$$\frac{\partial y}{\partial\eta}\frac{\partial T}{\partial t}=\frac{\lambda}{\rho c}\frac{\partial}{\partial\eta}\left(\frac{\frac{\partial T}{\partial\eta}}{\frac{\partial y}{\partial\eta}}\right)$$

The boundary conditions for T are :

on the curve $y_0(\xi,\eta)=0 \quad \lambda\frac{\partial T}{\partial\eta}=-q_w\frac{\partial y}{\partial\eta}$

and on the curve $\frac{1}{y_0(\xi,\eta)}=0 \quad T=T_\infty$

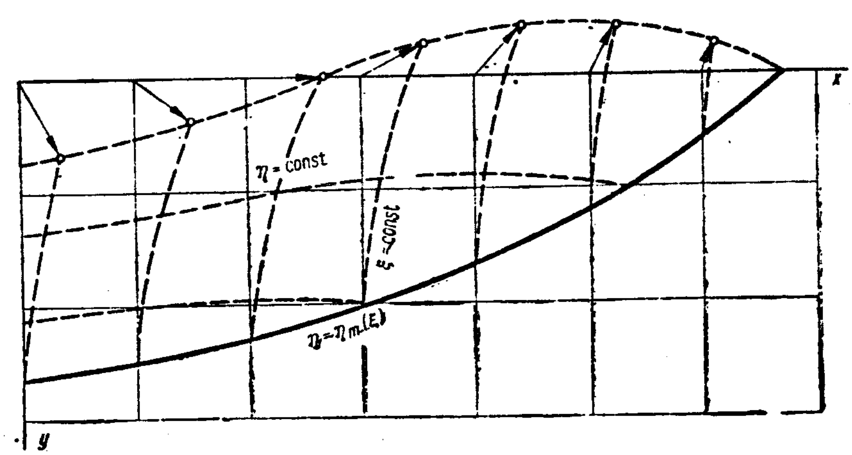

Fig. 1

Thus the problem has been reduced to the solution of equations (2.2), (2.4), (2.5) and (2.6). The equation (2.6) is resolved by the network method on lines ξ = const. The integrals (2.4) and (2.5) are computed over the time for each step, and then from the equations (2.2) $x(\xi,\eta,t)$, $y(\xi,\eta,t)$ are found in the following moment of time. Note that by the selection of $x_0(\xi,\eta)$, $y_0(\xi,\eta)$, we may succeed in obtaining that to a uniform net in coordinates ξ, η a nonuniform net in coordinates x, y correspond in the region of great temperature gradients with a small step,

DISTRIBUTION

GODDARD SPACE F.C.

600	TOWNSEND
	PIEPER
610	MEREDITH
611	McDONALD
612	HEPPNER
613	KUPPERIAN
614	LINDSAY, WHITE
615	BOURDEAU, BAUER
640	HESS [3]
	O'KEEFE
660	GI for SS [5]
630	MATTHEWS
633	KIDWELL
670	BAUMANN
673	LeDOUX
620	MAZUR
252	LIBRARY [5]
256	FREAS

NASA HQS.

SS	NEWELL, CLARK
SE	GARBARINI
SP	STANSELL
SG	NAUGLE
	SCHARDT
	ROMAN
	DUBIN
SL	LIDDEL
	FELLOWS
	HIPSHER-HOROWITZ
SM	FOSTER
	TROMBKA
	BADGLEY
RR	KURZWEG
RRA	WILSON
RRM	DEUTSCH
RRP	GESSOW
RV-1	PEARSON
	DeMERITTE
RV-2	ROSCHE
ATSS	WOLKO
	SCHWIND
	ROBBINS
AO-4	

OTHER CENTERS

AMES R.C.

SONETT	[5]
94035	[3]

LANGLEY R.C.

161	TRIMPI [3]
213	KATZOFF
	DAVIS [3]
242	GARRICK
303	ROBERTS
338	BRASLOW
185	WEATHERWAX [3]
188	ANDERSON

Contract No.NAS-5-3760
Consultants & Designers, Inc.
Arlington, Virginia

Translated by ANDRE L. BRICHANT
4 November 1964
